

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

**Math 10560, Exam 3.**  
**April 22, 2008**

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!					
1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
.....					
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
.....					
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
.....					
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
.....					
9.	(a)	(b)	(c)	(d)	(e)

<b>Please do NOT write in this box.</b>	
Multiple Choice	_____
10.	_____
11.	_____
12.	_____
13.	_____
Total	_____

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

Multiple Choice

1.(6 pts.) Compute the following limit:

$$\lim_{n \rightarrow \infty} \frac{\sin n}{n^2}$$

- (a)  $\infty$                       (b) 1                      (c) 0  
(d)  $\sin n$                       (e) Does not exist

2.(6 pts.) Compute the following limit:

$$\lim_{n \rightarrow \infty} \frac{3n^2(n-2)!}{n!}$$

- (a) 0                      (b) 1                      (c)  $\infty$   
(d) 3                      (e)  $-\infty$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

3.(6 pts.) Does the series  $\sum_{n=0}^{\infty} \frac{3+2^n}{\pi^{n+1}}$  converge or diverge? If it converges, compute its value.

- (a) converges to  $\frac{1}{\pi-3} + \frac{1}{\pi-2}$       (b) converges to  $\frac{3}{1-\pi} + \frac{2}{2-\pi}$   
(c) converges to  $\frac{3\pi}{\pi-1} + \frac{\pi}{\pi-2}$       (d) diverges  
(e) converges to  $\frac{3}{\pi-1} + \frac{1}{\pi-2}$

4.(6 pts.) Which of the following statements are true about the series  $\sum_{n=1}^{\infty} \frac{n^2+1}{n^5-n^2\sqrt{3}}$ ?

I. This series converges because  $\lim_{n \rightarrow \infty} \frac{n^2+1}{n^5-n^2\sqrt{3}} = 0$ .

II. This series converges by Ratio Test.

III. This series converges by Limit Comparison Test against the p-series  $\sum_{n=1}^{\infty} \frac{1}{n^3}$ .

- (a) II, III only      (b) I, II only      (c) III only  
(d) None      (e) I, III only

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

5.(6 pts.) One of the statements below holds for the series  $\sum_{n=1}^{\infty} \frac{\cos(2n)}{n^2 + 1}$ . Which one?

- (a) This series diverges by Ratio Test.
- (b) This series is conditionally convergent.
- (c) This series converges by Alternating Series Test.
- (d) This series is absolutely convergent by Comparison Test.
- (e) This series diverges because  $\lim_{n \rightarrow \infty} \frac{\cos(2n)}{n^2 + 1}$  is not 0.

6.(6 pts.) Which of the following statements are true about the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ ?

- I. This series converges by the Alternating Series Test.
- II. This series converges by the Ratio Test.
- III. This series converges absolutely.

- (a) I, III only
- (b) I, II only
- (c) I, II, III
- (d) II, III only
- (e) None

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

7.(6 pts.) Compute the radius of convergence of the power series  $\sum_{n=1}^{\infty} 2^n(x-1)^{2n}$ .

- (a)  $R = \frac{1}{2}$                       (b)  $R = 2$                       (c)  $R = \infty$   
(d)  $R = \frac{\sqrt{2}}{2}$                       (e)  $R = 0$

8.(6 pts.) Identify the Taylor Series of  $f(x) = \sin(x)$  centered at  $a = \frac{\pi}{2}$ , and its interval of convergence.

- (a)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ ,  $I = [-1, 1)$   
(b)  $\sum_{n=0}^{\infty} \frac{(-1)^n (x - \pi/2)^{2n}}{(2n)!}$ ,  $I = (-\infty, \infty)$   
(c)  $\sum_{n=0}^{\infty} \frac{(-1)^n (x - \pi/2)^{2n+1}}{(2n+1)!}$ ,  $I = (-\infty, \infty)$   
(d)  $\sum_{n=0}^{\infty} \frac{(-1)^n (x + \pi/2)^{2n}}{(2n)!}$ ,  $I = \left[\frac{\pi}{2} - 1, \frac{\pi}{2} + 1\right)$   
(e)  $\sum_{n=0}^{\infty} \frac{(-1)^n (x - \pi/2)^{2n+1}}{2n+1}$ ,  $I = \left[\frac{\pi}{2} - 1, \frac{\pi}{2} + 1\right)$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

9.(6 pts.) The following is the fourth order Taylor polynomial of the function  $f(x)$  at  $a$ .

$$T_4(x) = 10 + 5(x - a) + \sqrt{3}(x - a)^2 + \frac{1}{2\pi}(x - a)^3 + 17e(x - a)^4$$

What is  $f'''(a)$ ?

- (a)  $\frac{1}{2\pi}$       (b)  $2\sqrt{3}$       (c)  $\frac{3}{\pi}$       (d)  $\frac{1}{6\pi}$       (e)  $17e$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

Partial Credit

You must show your work on the partial credit problems to receive credit!

10. (10 pts.) a) (5 pts) Give a power series representation for  $e^{x^2}$ .

b) (5 pts) Find the limit

$$\lim_{x \rightarrow 0} \frac{e^{x^2} - 1 - x^2}{x^4}.$$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

11. (12 pts.) Consider the function  $f(x) = \frac{1}{2 - 3x}$ .

a) (4 pts.) Find the Taylor series of  $f(x)$  centered at 0.

b) (3 pts.) Determine the radius of convergence of this power series.

c) (4 pts) Find a power series representation for  $\frac{1}{(2 - 3x)^2}$  and give its radius of convergence.

d) (1pt) What is the value of the series you found in part (c) at  $x = 1/2$ ?



Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

**12.** (12 pts.) Find the interval of convergence of the following power series:

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{n}$$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

**13.** (12 pts.) Use the Integral Test to determine whether the series

$$\sum_{n=2}^{\infty} \frac{\ln(n)}{n^3}$$

is divergent or convergent. You must show that the Integral Test can be used in this situation.

**Note:** A correct answer with no work is worth only 3 points.

**Hint:** Use Integration By Parts.

Name: \_\_\_\_\_

Instructor: ANSWERS

**Math 10560, Exam 3.**

**April 22, 2008**

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!				
1.	(a)	(b)	(●)	(d) (e)
2.	(a)	(b)	(c)	(●) (e)
.....				
3.	(a)	(b)	(c)	(d) (●)
4.	(a)	(b)	(●)	(d) (e)
.....				
5.	(a)	(b)	(c)	(●) (e)
6.	(●)	(b)	(c)	(d) (e)
.....				
7.	(a)	(b)	(c)	(●) (e)
8.	(a)	(●)	(c)	(d) (e)
.....				
9.	(a)	(b)	(●)	(d) (e)

<b>Please do NOT write in this box.</b>	
Multiple Choice	_____
10.	_____
11.	_____
12.	_____
13.	_____
Total	_____